Numerical experiments did show that it is better to use a gradual variation of the porosity factor going from $\bar{\sigma}=0$ on the solid region to $\bar{\sigma}=\bar{\sigma}_{max}$ on the maximum porosity region of the airfoil. Thus, the following porosity distribution function was adopted:

$$\bar{\sigma}=\bar{\sigma}_{max}\sqrt{\sin\left(\pi\frac{x-x_1}{x_2-x_1}\right)} \tag{9}$$

In order to obtain the desired pressure distribution on the airfoil, any porosity distribution function may, in principle, be used. This also makes the method appropriate as a design procedure.

The graphically represented pressure distributions show that on the porous airfoil the flow is shockless. This important feature is maintained over a quite large Mach number domain as can be seen from the C_D-C_F curves plotted against Mach number in Fig. 4 for the same porosity distribution. A sensible increase in the Mach drag rise number is visible for the porous airfoil, especially in the case of a ventilated cavity. In the numerical experiments, an important fact was observed: for the subcritical Mach numbers the influence of the porosity on the pressure distribution is negligible.

These theoretical results, obtained in the small-disturbance, steady, inviscid flow theory are promising. They must be verified with a more complete flow model (including viscosity effects) and especially by comparison with experimental data.

References

[1]Bauer, F., Garabedian, F., and Korn, D., "Supercritical Wing Sections, III," *Lecture Notes in Economics and Mathematical Systems*, No. 150, Springer-Verlag, Berlin, Heidelberg, New York, 1977, pp. 82-83.

[2]Dumitrescu, L. Z. and Savu, G., "Influence de l'écoulement dans les chambres plénum sur les corrections d'essais en souffleries transsoniques," IMFCA Report, 1977.

[3]Carafoli, E., "On a Permeable Lifting Surfaces Theory," *Lucrarile Sesiunii Generale Stiintifice din 2-12 iunie 1950*, Editura Academiei, Romanian Peoples Republic, 1951.

[4]Rahmatulin, H. A., "Perforated Aerodynamic Shapes," *Vestnik Moskovskovo Universiteta*, No. 3, 1950, p. 41.

[5]Patraulea, N. N., "Aerodynamics of the Porous Surfaces," Editura Academiei, Romanian Peoples Republic, 1956, pp. 26-32.

[6]Ventres, C. S. and Barakat, R., "Aerodynamics of Airfoils With Porous Trailing Edges," *The Aeronautical Quarterly*, Vol. XXX, Pt. 2, May 1979, pp. 387-399.

[7]Ovchinnikov, O. N., "The Perforated Flat Plate in Stabilized Viscous Flow," *Mechanika Zhidkosti i Gaza*, Nov.-Dec. 1966, pp. 65-73.

[8]Savu, G., "A Computer Program for 2D Transonic Flow in Small Disturbances Theory Calculation 222SG58A," INCREST, 1981.

[9]Murman, E. M. and Cole, J. D., "Calculation of Plane Steady Transonic Flow," *AIAA Journal*, Vol. 9, Jan. 1971, pp. 114-121.

Higher Order Strip Integral Method for Three-Dimensional Boundary Layers

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Nomenclature

I_j = influence coefficients
N = number of strips
R_e = Reynolds number based on velocity outside the boundary layer
U,V = X and Y components of velocity outside the boundary layer
u,v = X and Y components of velocity inside the boundary layer
X,Y,Z = coordinate directions
η = Z/δ_{11} or Z/δ_{22}; nondimensional coordinate in Z direction
δ_{11},δ_{22} = boundary-layer thicknesses in the X and Y directions, respectively
δ_1,δ_2 = displacement thicknesses along and normal to external streamline direction
θ_1,θ_2 = momentum thicknesses along and normal to external streamline direction

Superscript

$(\ )'$ = differentiation with respect to η

Subscripts

$,(\ )$ = derivative with respect to the accompanying quantity
e = reference quantity at edge of boundary layer
n = component perpendicular to external flow
t = component parallel to external flow
L = upper bound of integration
∞ = reference quantity at freestream

Introduction

THE design of aircraft wings and other similar lifting surfaces is highly influenced by the boundary layer over the surface. Two-dimensional boundary layers can be calculated relatively easily. Computation of three-dimensional boundary layers is difficult and time consuming. A recent study by Humphreys[1] showed that the current methods for the computation of three-dimensional boundary layers lack accuracy, even when the cross flow is small. Yet another study by Lemmerman[2] showed that there exists a considerable degree of scatter in the results by finite difference methods. Three-dimensional computations usually take large computational time, especially if finite difference methods are used. In practical applications the boundary-layer codes form an intermediate part of a much larger program along with a potential flow code. An iterative solution between these two analyses utilizes a large amount of computing resources before convergence. The integral methods similar to that by Stock[3] are more attractive than finite difference methods in this respect. These methods are based on parametric representation of velocity profiles and are expected to give good results when the real flowfield is a subset of the chosen class of assumed profiles. Three-dimensional boundary layers consist of a wide variety of profiles, and parametric representation may not always be possible. Strip integral methods offer an alternative to solve this problem. The boundary layer can be assumed to be divided into a few strips with the velocity profiles varying according to a simple power law between the strips. This approach provides a large degree of flexibility by eliminating the need for assumed profiles. Moreover, the velocity profiles can be predicted, if needed, by choosing a large number of strips. A variety of boundary layers can be studied with a wide spectrum of differing velocity profiles. The more complex the flow, the greater the number of strips required for computation.

In order to study the feasibility of such an approach, a laminar flow problem with large cross flows was chosen. Loos[4] has solved the three-dimensional laminar boundary layer formed due to parabolic streams over a flat plate. The results obtained by the application of strip integral method to the above problems is found to be very encouraging. The details of the method are briefly described below.

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Higher Order Strip Integral Method (HOSIM)

The basic method is described in detail by Belotserkovskii and Chuskin,[5] with only the essential details given herein. The boundary-layer equations are written for a rectangular coordinate system. The X and Y directions lie on the surface of a planar wing. The Z direction is assumed to be perpendicular to the above plane. The momentum equations are normalized with respect to reference quantities such as external flow velocity outside of the boundary layer, reference chord, and the local boundary-layer thicknesses. Different boundary-layer thicknesses are assumed for each of the momentum equations. The equations are written in an integral form by introducing an integration in the Z direction. An arbitrary number of equations can be generated from this integral representation by choosing different upper or lower bounds for the integrals. The approach followed herein is to keep the lower bound as that corresponding to the surface and to vary the upper bound. Since the equations are normalized with respect to boundary-layer thickness, the nondimensional Z coordinate will have an upper bound at unity. The region below this upper bound is divided into a number of strips which are not necessarily of the same width. One pair of equations is obtained by integrating the X and Y momentum equations from the surface to the upper boundary of each strip. The continuity equation is utilized to eliminate the terms involving the Z component of velocity. It is convenient to assume the strips in such a manner that the upper boundaries are functions independent of either the X or Y coordinate. In such an approach the integral and the differential operators can be mutually interchanged without introducing additional terms in the equations. The resulting equations are shown below.

$$\int_0^{\eta_L} (2uu_{,X} - u_L u_{,X})\,d\eta + \frac{U_{\infty,X}}{U_\infty}\int_0^{\eta_L}(2u^2 - u_L u)\,d\eta + \frac{\delta_{11,X}}{\delta_{11}}\int_0^{\eta_L}(u^2 - u_L u)\,d\eta = \frac{1}{R_e\delta_{11}^2}\,[U_{,\eta}]\Big|_0^{\eta_L} + \eta_L\left[\frac{U_e U_{e,X}}{U_\infty^2} + \frac{V_e U_{e,X}}{U_\infty^2}\right] \tag{1}$$

$$\int_0^{\eta_L} (uv_{,X} + u_{,X}v - v_L u_{,X})\,d\eta + \frac{U_{\infty,X}}{U_\infty}\int_0^{\eta_L}(2uv - v_L u)\,d\eta + \frac{\delta_{22,X}}{\delta_{22}}\int_0^{\eta_L}(uv - v_L u)\,d\eta = \frac{1}{R_e\delta_{22}^2}\,[v_{,\eta}]\Big|_0^{\eta_L} + \eta_L\frac{U_e V_{e,X}}{U_\infty^2} \tag{2}$$

In order to convert this integro-differential equation into a differential equation, some assumptions have to be made to evaluate the integrals. In the present approach, it is assumed that the velocity varies according to a second-order polynomial between the strip boundaries. With the above assumption, the equations can be integrated. The resulting equations will be differential equations in terms of unknown velocities at strip boundaries and of unknown boundary-layer thicknesses. The integration process introduces influence coefficients associated with these velocities. These influence coefficients are functions of strip widths and, in the present scheme, can be evaluated, once and for all, at the beginning of the computation. The resulting equations are shown below.

$$\sum_{j=1}^{N} I_j\left\{(2u_{j+1}u_{j+1,X} - u_{i+1}u_{j+1,X}) + \frac{U_{\infty,X}}{U_\infty}(2u_{j+1}^2 - u_{i+1}u_{j+1}) + \frac{\delta_{11,X}}{\delta_{11}}(u^2{}_{j+1} - u_{i+1}u_{j+1})\right\} = \frac{1}{R_e\delta_{11}^2}(u_{i+1,\eta} - u_{1,\eta}) + \eta_{i+1}\left(\frac{U_e U_{e,X}}{U_\infty^2} - \frac{V_e V_{e,y}}{U_\infty^2}\right) \tag{3}$$

$$\sum_{j=1}^{N} I_j\left\{(u_{j+1}v_{j+1,X} + u_{j+1,X}v_{j+1} - v_{i+1}u) + \frac{U_{\infty,X}}{U_\infty}(2u_{j+1}v_{j+1} - v_{i+1}u_{j+1}) + \frac{\delta_{22,X}}{\delta_{22}}(u_{j+1}v_{j+1} - v_{i+1}u_{j+1})\right\} = \frac{1}{R_e\delta_{22}^2}(v_{i+1,\eta} - v_{1,\eta}) + \eta_{i+1}\frac{U_e V_{e,X}}{U_\infty^2} \tag{4}$$

Since the velocities on the surface and at the edge of the boundary layer are all known, there are as many equations as there are unknown derivatives with respect to one of the coordinate directions. If the initial values are known along the Y direction, then the Y derivatives can be evaluated numerically. The system of linear equations can be solved to obtain the value of X derivatives. These X derivatives can then be used for integrating the equations in the X direction.

Flat Plate with Large Cross Flow

In order to study a case with considerable cross flow, a flat plate problem with parabolic external streamlines was con-

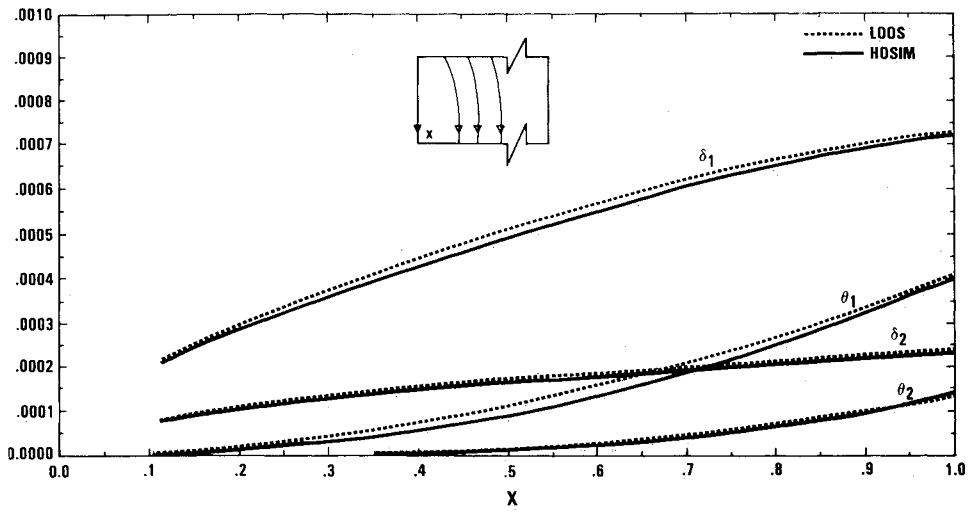

Fig. 1 Comparison of results by HOSIM and by similarity solution.

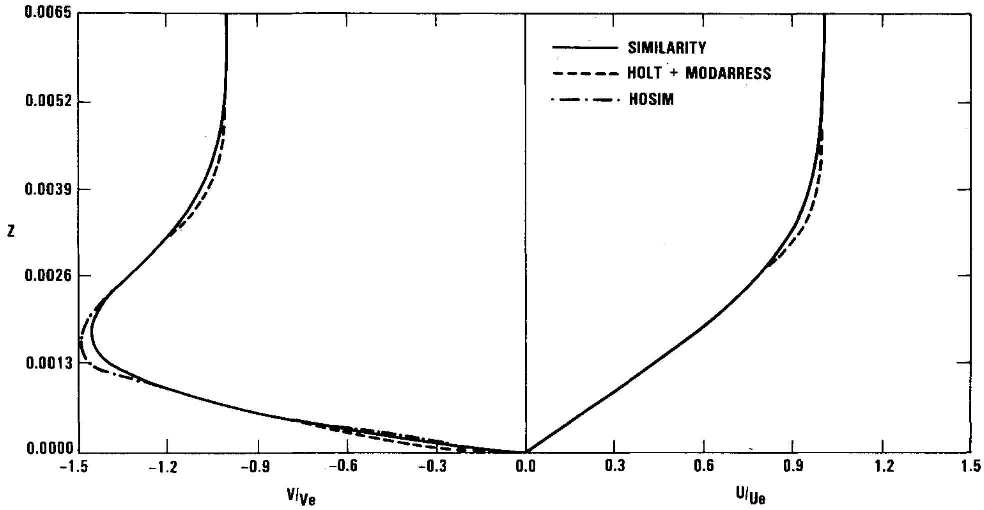

Fig. 2 Velocity profiles inside the boundary layer.

sidered. The external streamlines are given by the following expression.

$$U_e = \text{const}, \quad V_e = U_e(1 - CX) \tag{5}$$

where C is an arbitrary constant.

Loos has solved this problem using similarity considerations and has given the following expressions for velocities tangential and normal to the external streamline directions.

$$v_t = U_e\{1 + (1-CX)^2\}^{-0.5}\{0.5f' + (1-CX)(0.5f' - CXh)\}$$

$$v_n = U_e CX\{1 + (1-CX)^2\}^{-0.5}\{h - 0.5f'\} \tag{6}$$

where f and h are given by the following differential equations and boundary conditions.

$$f''' + ff'' = 0, \quad h'' + fh' - 2f'h + 4 = 0 \tag{7}$$

with the boundary conditions,

$$f' = f = h = 0 \quad \text{when} \quad \eta = 0$$

$$f' \to 2 \text{ and } h \to 1 \quad \text{when} \quad \eta \to \infty \tag{8}$$

A higher order strip integral approach was applied to this problem. The initial values required for starting the solution were obtained from the above expression. A Runge-Kutta-Gill procedure with automatic step-size control was used for integrating the equations. A cubic spline interpolation technique was used to interpolate the velocities at strip boundaries.

Results

Figure 1 shows the comparison of results obtained by HOSIM and by the approach by Loos. The latter method can be considered as an exact method. The results show that the present approach has the capability to predict the boundary-layer quantities, even in the case of large cross flows. It can analyze flowfields involving a wide spectrum of boundary-layer profiles. Choice of a large number of strips would be similar to finite-difference analysis. But good results can be obtained by using a relatively small number of strips. The minimum number of strips required to produce reliable answers will be more for complex profiles compared to that for simple two-dimensional profiles. Five strips were used in this analysis, which produced the results shown in Fig. 1. Figure 2 shows the velocity profiles calulated by HOSIM plotted against the results by Holt and Modarress.[6] In both cases the initial values are obtained from the similarity solution. The X component of velocity was predicted almost exactly by HOSIM. The apparent greater accuracy of Fig. 2 is due to the reason that the initial conditions were applied at a location closer to the computational position than that for Fig. 1.

References

[1]Humphreys, D. A., "Three-Dimensional Wing Boundary Layer Calculated with Eight Different Methods," *AIAA Journal*, Vol. 19, Feb. 1981, pp. 232-234.

[2]Lemmerman, L.A. and Atta, E. W., "A Comparison of Existing Three Dimensional Boundary Layer Calculation Methods," AIAA Paper 80-0133, Jan. 1980.

[3]Stock, H.W.,"Computation of the Boundary Layer and Separation Lines on Inclined Ellipsoids and of Separated Flows on Infinite Swept Wing," AIAA Paper 80-1442, July 1980.

[4]Loos, H.G.,"A Simple Laminar Boundary Layer with Secondary Flow," *Journal of Aerospace Sciences*, Jan. 1955, pp. 35-40.

[5]Belotserkovskii, O. M. and Chushkin, P.I., "The Numerical Solution of Problems in Gas Dynamics," *Basic Developments in Fluid Dynamics*, Vol. 1,edited by M. Holt, 1965, pp. 1-23.

[6]Holt, M. and Modarress, D. "Application of the Methods of Integral Relations to Laminar Boundary Layers in Three Dimensions," *Proceedings of Royal Society of London*, Ser. A. Vol. 353, 1977, pp. 319-347.

A Flutter Eigenvalue Program Based on the Newton-Raphson Method

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Introduction

SINCE the elements of the matrix of the Laplace transformed equations of motion[1] for an elastic wing or aircraft are nonlinear functions of the transform parameter p ($= ik$, k being the reduced frequency), the eigenvalue problem associated with these equations is a nonlinear eigenvalue problem. In a program by Stark,[2] this problem was solved by iterative application of a routine for solution of a linear eigenvalue problem, while other authors[3,4] first introduced

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